**PROJECT III - MATH 800**
**DUE APRIL 6, 2021**

(1) Problem 2/page 145;
(2) Problem 5 b), c), f) /page 145;
(3) Problem 8 a)/page 146;
**Hint:** Consider $g(z) = f(z)(z - P)$, defined for $D(P, r) \setminus \{P\}$. Write its Taylor series (why does it exists?) and argue from there.
(4) Problem 9/page 146;
**Hint:** Show first that what is asked follows from: If $f$ has essential singularity at $P$, then for every $n$

$$\limsup_{z \to P} |(z - P)^n f(z)| = \infty.$$

Then, work to show this last formulation by contradiction.
(5) Problem 13 b), c), e)/page 147;
(6) Problem 14/page 147;
**Hint:** Answer is no. Try the sequence of partial sums of a Laurent series at a essential singularity like $e^{1/z}$.
(7) Problem 38/page 152.
(8) 47/page 154
**Hint:** Consider

$$\int_{\gamma_R} \frac{e^{iz}}{1 + z^4} dz.$$

(9) 51/page 154
**Hint:** Note

$$\int_0^\infty \frac{x \sin(x)}{1 + x^2} dx = \frac{1}{2} \int_{-\infty}^\infty \frac{x \sin(x)}{1 + x^2} dx.$$

(10) 55/page 154.
**Hint:** Consider on a contour like the one on top of p. 133.

$$\int_{\gamma_{R,\epsilon}} \frac{1 - e^{2iz}}{2z^2} dz$$