PROJECT I - MATH 800
SPRING 2021
DUE ON FEBRUARY, 25

(1) Problem 19/page 23;
(2) Problem 29 a), b)/page 24
(3) Problem 34/page 25;
(4) Problem 47/page 26;
Hint: For $f = u + iv$, write $\ln(|f|) = \frac{1}{2}\ln(u^2 + v^2)$. Take derivatives and use the Cauchy-Riemann for u, v.
(5) Problem 52/page 27
Hint: To show the non-existence of a holomorphic f, so that $f : \frac{\partial f}{\partial z}(z) = \frac{1}{z}$ argue by contradiction, by considering the path integral
$$\int_{|z|=1} \frac{\partial f}{\partial z} dz = \int_{|z|=1} \frac{1}{z} dz$$
(6) Prove that if a holomorphic function F on a connected domain satisfies $F'(z) = 0$, then $F = const.$
Hint: We have done similar result in class for real-valued C^1 functions.
(7) Let $\Omega \subset \mathbb{C}$ be an open set and $F \in C^2(\Omega) \cap H(\Omega)$. Show that $\frac{\partial F}{\partial z} \in H(\Omega)$
Hint: Cauchy-Riemann
(8) Problem 55/page 27 without the counterexample.
Hint: You have that on the connected intersection $U_1 \cap U_2$, there is $F_1' = F_2'$. Use the result in Exercise 6.
(9) Problem 37/page 66;
(10) Problem 42/page 66;
Hint:
The hint to Exercise 3 is misleading. Instead,
- Show that the function
$$F(z) := \int_{\gamma_z} f(\xi) d\xi$$
is defined correctly, when γ is any smooth curve connecting 0 to z. That should use the condition that $\oint_\sigma f(\xi) d\xi = 0$ for any closed curve σ.
- Show that the the function $F(z) = \int_{\gamma_z} f(\xi) d\xi = \int_{\gamma_z} f(\xi) dx + i \int_{\gamma_z} f(\xi) dy$ satisfy $F_x = f, F_y = if$, which imply the Cauchy-Riemann for F. Here, you need to take the curves γ_z appropriately, so that you can produce the difference quotient for F_x, F_y like we did in the various proofs for the solvability of the system $u_x = f, u_y = g$.